Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 18 May 2023

Afternoon (Time: 2 hours)

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary
PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. A curve has equation

$$y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

(b) Hence find the range of values of x for which y is decreasing.

(4)

1 (a) $y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$ (1)

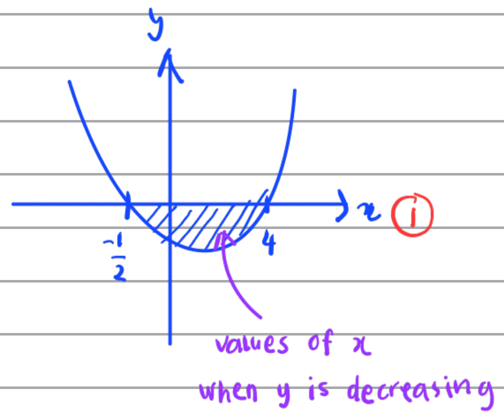
$\frac{dy}{dx} = 2x^2 - 7x - 4$ (1)

(b) $\frac{dy}{dx} = 0$

$2x^2 - 7x - 4 = 0$

$(2x + 1)(x - 4) = 0$ (1)

values of x : $-\frac{1}{2}$, 4 (1)



$-\frac{1}{2} < x < 4$ (1)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = \sqrt{x}$ or otherwise, solve

$$6x + 7\sqrt{x} - 20 = 0$$

(4)

$u = \sqrt{x}$

$x = u^2$ (1)

$6x + 7\sqrt{x} - 20 = 0$

Substitute u^2 in place of x :

$6u^2 + 7u - 20 = 0$

$(3u - 4)(2u + 5) = 0$ (1)

$u = \frac{4}{3}$, $u = -\frac{5}{2}$

$\sqrt{x} = \frac{4}{3}$, $\sqrt{x} = -\frac{5}{2}$ (1)

$x = \frac{16}{9}$ only (1)

square root of x can not be negative

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Question 2 continued

(Total for Question 2 is 4 marks)

3.

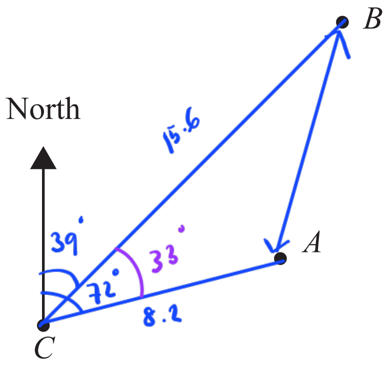


Figure 1

Figure 1 is a sketch showing the position of three phone masts, A, B and C.

The masts are identical and their bases are assumed to lie in the same horizontal plane.

From mast C

- mast A is 8.2 km away on a bearing of 072°
- mast B is 15.6 km away on a bearing of 039°

(a) Find the distance between masts A and B, giving your answer in km to one decimal place.

(3)

An engineer needs to travel from mast A to mast B.

(b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels.

(1)

a) $\angle BAC$ = bearing A - bearing B

$= 72° - 39° = 33°$ (1)

Cosine rule : $A^2 = B^2 + C^2 - 2BC \cos A$

Using cosine rule to get the length AB :

$AB^2 = 15.6^2 + 8.2^2 - 2(15.6)(8.2)\cos 33°$ (1)

$AB^2 = 96.03$

$AB = 9.8$ km (1)

Question 3 continued

b) In a real case scenario, the road is unlikely to be completely straight. Therefore, the distance AB is likely to be longer. 1

(Total for Question 3 is 4 marks)

4. (a) Sketch the curve with equation

$$y = \frac{k}{x} \qquad x \neq 0$$

where k is a positive constant.

(2)

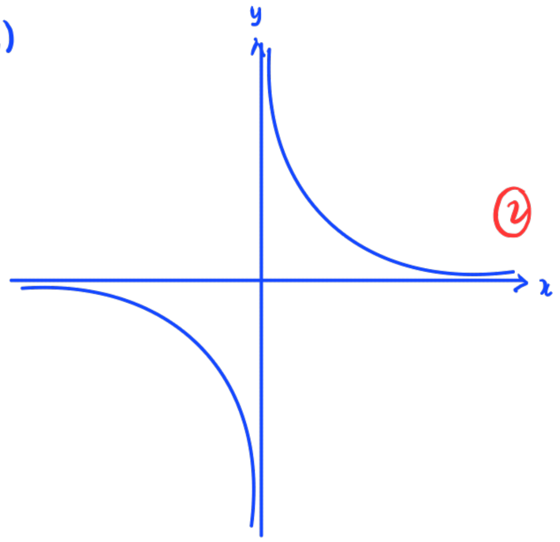
(b) Hence or otherwise, solve

$$\frac{16}{x} \leqslant 2$$

(3)

Tips : if you don't know where to start, always consider putting values into the equation.

a)



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if $x = 0,\ y = \infty$

$y = 0,\ x = \infty$

hence, the curve should not touch both x and y axis.

b) $\frac{16}{x} = 2$, $x < 0$ 1

substitute into the equation, when $x<0$, the value on RHS will always be ≤ 2. So, $x<0$ is a solution.

$x = \frac{16}{2} = 8$

x must be ≥ 8 because $x<0$ is already considered above.

Hence,

$\therefore\ x < 0\ ,\ x \geq 8$ 1

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Question 4 continued

(Total for Question 4 is 5 marks)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

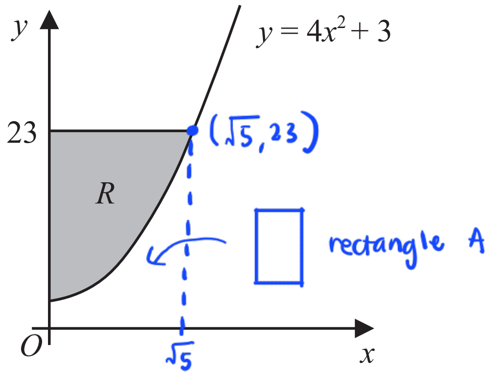

Figure 2

The finite region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 4x^2 + 3$, the y-axis and the line with equation $y = 23$

Show that the exact area of R is $k\sqrt{5}$ where k is a rational constant to be found.

(5)

Finding intersection point between the curve and line $y = 23$:

$23 = 4x^2 + 3$

$x^2 = 5$

$x = \sqrt{5}$ (1)

intersection point is $(\sqrt{5}, 23)$

Finding area bounded by the curve :

$\int_0^{\sqrt{5}} 4x^2 + 3 = \left[\frac{4}{3}x^3 + 3x\right]_0^{\sqrt{5}}$ (1)

$= \frac{4}{3}(\sqrt{5})^3 + 3\sqrt{5}$ (1)

Finding area of rectangle A :

$A = 23 \times \sqrt{5} = 23\sqrt{5}$

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Question 5 continued

Finding area of shaded region R :

: Area of rectangle A - area bounded by the curve

$= 23\sqrt{5} - \left(\frac{4}{3}(\sqrt{5})^3 + 3\sqrt{5}\right)$ 1

$= \frac{40}{3}\sqrt{5}$ 1

(Total for Question 5 is 5 marks)

6. The circle C has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

Given that C does not cut or touch the x-axis,

(b) find the range of possible values for k.

(3)

a) $x^2 + y^2 - 6x + 10y + k = 0$

$x^2 - 6x + y^2 + 10y + k = 0$

$(x-3)^2 + (y+5)^2 - 9 - 25 + k = 0$ (1)

centre of C = $(3, -5)$ (1)

b) If circle touch the x-axis, $y = 0$

Hence, $x^2 - 6x + k = 0$.

However, we know that C does not touch the x-axis.

So, $b^2 - 4ac < 0$

$(-6)^2 - 4(1)(k) < 0$

$4k > 36$

$k > 9$. (1)

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Question 6 continued

Radius should be > 0, so :

$(x-3)^2 + (y+5)^2 = \boxed{9 + 25 - k}$ ← radius

$9 + 25 - k > 0$ (1)

$k < 34$

Combine the two solutions :

$\therefore \quad 9 < k < 34$ (1)

(Total for Question 6 is 5 marks)

7. The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km

Using a **linear model**, with V litres being the volume of fuel remaining in the car's fuel tank and d km being the distance the car had travelled,

(a) find an equation linking V with d.

(4)

Given that, on a particular journey

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

find, according to the model,

(b) (i) the initial volume of fuel that was in the fuel tank of the car,

(ii) the distance that the car travelled on this journey.

(3)

In fact the car travelled 320 km on this journey.

(c) Evaluate the model in light of this information.

(1)

a) for equation of linear model V and d,

$V = ad + b$ 1, where a and b are unknown.

After 80 km journey :

$40 = 80a + b$ — ① 1

After 200 km journey :

$25 = 200a + b$ — ② 1

substitute ② into ①

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Question 7 continued

$40 = 80a + (25 - 200a)$

$120a = -15$

$a = -\frac{1}{8}$, $b = 50$

$\therefore V = -\frac{1}{8}d + 50$ 1

(b) (i) initial volume of the fuel is when $d = 0$.

$V = -\frac{1}{8}(0) + 50$

$= 50$ litres 1

(ii) Total distance travelled is when $V = 0$

$0 = -\frac{1}{8}d + 50$

$d = 50 \times 8$ 1

$= 400$ km

(c) when $d = 320$ km,

$V = -\frac{1}{8}(320) + 50$

$= 10$ litres

$\therefore$ Concludes that this is a poor model because 10 litres of fuel is significantly more than empty tank. 1

(Total for Question 7 is 8 marks)

8.

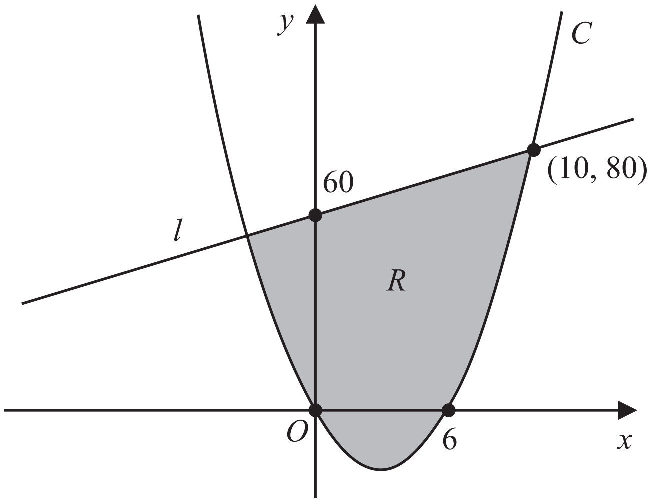

Figure 3

Figure 3 shows a sketch of a curve C and a straight line l.

Given that

- C has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic expression in x
- C cuts the x-axis at 0 and 6
- l cuts the y-axis at 60 and intersects C at the point $(10, 80)$

use inequalities to define the region R shown shaded in Figure 3.

(5)

has all values needed to form equation

Finding the equation of line l :

gradient of l : $\frac{80-60}{10} = 2$ 1

y-intercept : 60

Hence, $y = 2x + 60$ 1 — equation of l

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Question 8 continued

Finding equation of curve C :

$y = ax^2 + bx$

at $(6,0)$: $0 = 36a + 6b$ — ①

at $(10,80)$: $80 = 100a + 10b$ — ② (1)

from ①

$b = \frac{-36a}{6} = -6a$ — ③

Substitute ③ into ②

$80 = 100a + 10(-6a)$

$80 = 40a$

$a = 2$, $b = -12$

$y = 2x^2 - 12x$

$= 2x(x-6)$ (1) — equation of curve C

Region R : $2x(x-6) \leq y \leq 2x + 60$ (1)

(Total for Question 8 is 5 marks)

9. Using the laws of logarithms, solve the equation

$$2\log_5(3x-2) - \log_5 x = 2$$

(5)

$$2\log_5(3x-2) - \log_5 x = 2$$

$$\log_5(3x-2)^2 - \log_5 x = 2 \quad (1)$$

$$\log_5 \frac{(3x-2)^2}{x} = 2 \quad (1)$$

$$\frac{(3x-2)^2}{x} = 5^2 \quad (1)$$

$$9x^2 - 12x + 4 = 25x \quad (1)$$

$$9x^2 - 37x + 4 = 0$$

$$(x-4)(9x-1) = 0$$

$x = 4$ and $x = \frac{1}{9}$

substitute $x = \frac{1}{9}$ into the equation of log will give $\log_5$ of a negative number which is not correct. Hence, $x = \frac{1}{9}$ is not a solution.

$\therefore x = 4$ only (1)

Question 9 continued

(Total for Question 9 is 5 marks)

10.

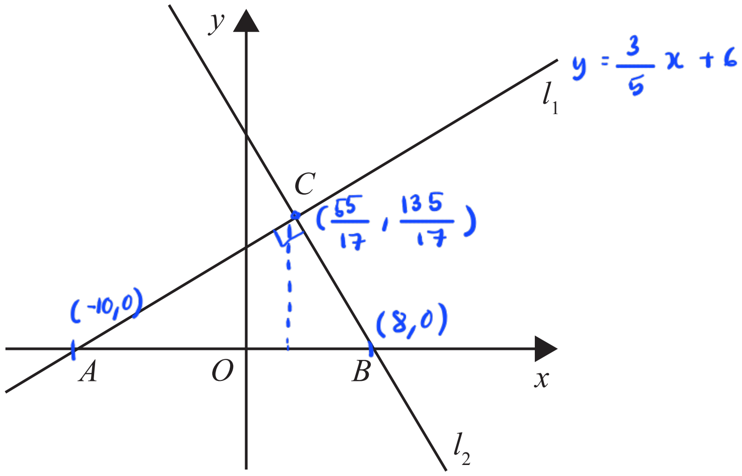

Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$5x + 3y = 40$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

a) l_1 : $y = \frac{3}{5}x + 6$, where gradient, $m_1 = \frac{3}{5}$

l_1 is perpendicular to l_2, therefore $m_1 = \frac{-1}{m_2}$

$m_2 = -\frac{5}{3}$ (1)

Since l_2 passes through $B(8,0)$, we can find equation of l_2 :

$y - 0 = -\frac{5}{3}(x - 8)$ (1)

$y = \frac{-5x}{3} + \frac{40}{3} \implies 5x + 3y = 40$ (1)

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Question 10 continued

b) Finding point A :

line l_1 cross the x-axis, so $y = 0$

$0 = \frac{3}{5}x + 6$

$x = -10 \quad \therefore A(-10, 0)$ (1)

Finding point C :

Solve equation l_1 and l_2 simultaneously to get point C.

$y = \frac{3}{5}x + 6$ — (1)

$5x + 3y = 40$ — (2)

Substitute (1) into (2)

$5x + 3\left(\frac{3}{5}x + 6\right) = 40$ (1)

$5x + \frac{9}{5}x + 18 = 40$

$\frac{34}{5}x = 22$

$34x = 110$

$x = \frac{55}{17}$ – substitute into (1)

$y = \frac{3}{\cancel{5}}\left(\frac{\cancel{55}^{11}}{17}\right) + 6 = \frac{135}{17}$ (1) $\quad \therefore C\left(\frac{55}{17}, \frac{135}{17}\right)$

length AB $\qquad$ height from x-axis to point C

Area of ABC $= \frac{1}{2} \times \text{length} \times \text{height} = \frac{1}{2} \times (8+10) \times \left(\frac{135}{17}\right)$ (1)

$= \frac{1215}{17}$ (1)

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

11. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \qquad t \in \mathbb{R} \quad t \geqslant 0$$

Using the model,

(a) find the height of the plant when it was first measured,

(2)

(b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year.

(3)

According to the model, there is a limit to the height to which this plant can grow.

(c) Deduce the value of this limit.

(1)

a) t years = years after first measured

first measured = 0 years

so, height when plant was first measured :

$h = 2.3 - 1.7e^{-0.2(0)}$ (1)

$= 2.3 - 1.7 = 0.6$ m (1)

calculating rate of change of height of the plant

b) $\frac{dh}{dt} = 0.34e^{-0.2t}$ (1)

when $t = 4$,

$\frac{dh}{dt} = 0.34e^{-0.2(4)}$ (1)

$= 0.153$ m ≈ 15.3 cm (1)

c) when t approaching ∞, the height will be 2.3 m. (1)

2.3 m is the value of the limit.

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Question 11 continued

(Total for Question 11 is 6 marks)

12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leqslant 360°$

$$4\tan x = 5\cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4\tan 3x = 5\cos 3x$$

in the interval $0 < x \leqslant 1800°$, explaining briefly the reason for your answer.

(2)

a) $4 \tan x = 5 \cos x$

$4 \dfrac{\sin x}{\cos x} = 5 \cos x$ 1

$4 \sin x = 5 \cos^2 x$

$4 \sin x = 5(1 - \sin^2 x)$ 1

$4 \sin x = 5 - 5 \sin^2 x$

$5 \sin^2 x + 4 \sin x - 5 = 0$ 1

Question 12 continued

b) $4\tan x = 5\cos x \equiv 5\sin^2 x + 4\sin x - 5 = 0$ (1)

Let $a = \sin x$, $5a^2 + 4a - 5 = 0$

$$a = \frac{-2 \pm \sqrt{29}}{5}$$

$$\sin x = \frac{-2 \pm \sqrt{29}}{5} \quad (1)$$

$$\boxed{\sin x = \frac{-2+\sqrt{29}}{5}}, \quad \frac{-2-\sqrt{29}}{5}$$

not a solution because the value is less than -1, so it is outside of range of $\sin x$

Sin	ALL
2nd	1st
3rd	4th
tan	cos

$$x = \sin^{-1} \frac{-2+\sqrt{29}}{5} \quad (1)$$

take the value from 2nd quadrant also since sin is +ve on this quadrant.

$= 42.6^\circ$ and $180^\circ - 42.6^\circ$
(1) $= 137.4^\circ$

c) for interval of $0^\circ < x \leq 360^\circ$,

The number of solution would be $2 \times 3 = 6$ because the x value decrease by factor of 3. (1)

for interval of $0 < x \leq 1800^\circ$, which is 5 times larger than the previous interval,

the number of solutions would be $6 \times 5 = 30$. (1)

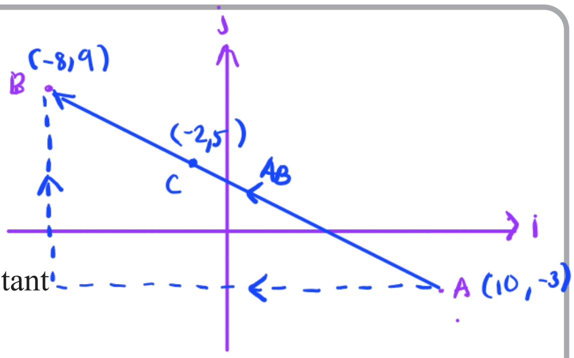
Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + {}^{5}p\mathbf{j}$ where p is a constant



(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

(2)

$\frac{12}{18} : \frac{2}{3}$

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(10\underset{\sim}{i} - 3\underset{\sim}{j}) + (-8\underset{\sim}{i} + 9\underset{\sim}{j})$ (1)

$= -10\underset{\sim}{i} - 8\underset{\sim}{i} + 3\underset{\sim}{j} + 9\underset{\sim}{j}$

$= -18\underset{\sim}{i} + 12\underset{\sim}{j}$ (1)

b) $\left|\overrightarrow{AB}\right| = \sqrt{(-18)^2 + (12)^2}$

$= \sqrt{468}$ (1)

$= \sqrt{36} \times \sqrt{13}$

$= 6\sqrt{13}$ (1)

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Question 13 continued

(c) (i) gradient BC = gradient BA (because all points are on the same line)

$$m_{BC} = \frac{q-p}{(-8)-(-2)} = \frac{q-p}{-6} \quad (1)$$

$$m_{BA} = \frac{q-(-3)}{(-8)-10} = \frac{12}{-18}$$

so, $\frac{q-p}{-6} = \frac{12}{-18}$

$3(q-p) = 12$

$27 - 3p = 12$

$3p = 15$

$p = 5$ (1)

(ii) since length AC : AB is 2 : 3,

ratio of triangle AOC is 2 : 3 to triangle AOB. (1)

Question 13 continued

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Question 13 continued

(Total for Question 13 is 7 marks)

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Turn over

14. Find, in simplest form, the coefficient of x^5 in the expansion of

$$\left(5 + 8x^2\right)\left(3 - \frac{1}{2}x\right)^6$$

(5)

Since the first term is $(5 + 8x^2)$, we need term of x^5 and x^3 from the second term to find the coefficient of x^5 from the expansion.

: $5 \times x^5$ term

: $8x^2 \times x^3$ term

x^5 term : ${}^6C_5 \times 3^1 \left(-\frac{1}{2}x\right)^5$ (1) $= -\frac{9}{16}x^5$

x^3 term : ${}^6C_3 \times 3^3 \left(-\frac{1}{2}x\right)^3 = -\frac{135}{2}x^3$ (1)

coefficient of x^5 in the expansion :

$\left(5 \times -\frac{9}{16}x^5\right)$ (1) $+ \left(8x^2 \times -\frac{135}{2}x^3\right)$ (1)

$= -\frac{45}{16}x^5 + \left(-540x^5\right)$

$= \left(-\frac{45}{16} - 540\right)x^5$

$= \frac{-8685}{16}$ (1)

Question 14 continued

(Total for Question 14 is 5 marks)

15. **In this question you must show detailed reasoning.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$

The curve C_2 has equation $y = x^2 - 12x + 14$

(a) Verify that when $x = 1$ the curves C_1 and C_2 intersect. **(2)**

The curves also intersect when $x = k$.

Given that $k < 0$

(b) use algebra to find the exact value of k. **(5)**

a) Substitute $x=1$ into both curve equations :

$y = 8 - 10(1) + 6(1)^2 - (1)^3 = 3$

$y = (1)^2 - 12(1) + 14 = 3$ 1

C_1 and C_2 meet at $(1,3)$, so they both intercept at $x=1$.

1

b) when curves intersect :

$8 - 10x + 6x^2 - x^3 = x^2 - 12x + 14$ 1

$x^3 - 5x^2 - 2x + 6 = 0$

$$\begin{array}{r}
x^2 - 4x - 6 \\
x-1 \,\big)\, \overline{x^3 - 5x^2 - 2x + 6} \\
-\; x^3 - x^2 \quad\quad\quad\quad \\
\hline
-4x^2 - 2x \quad\quad \\
-\; -4x^2 + 4x \quad\quad \\
\hline
-6x + 6 \\
-\; -6x + 6 \\
\hline
\end{array}$$

from (a), $(x-1)$ is a factor of the cubic. Hence,

$(x-1)(x^2 - 4x - 6) = 0$

1 1

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Question 15 continued

By using quadratic formula :

$(x-1)\boxed{(x^2-4x-6)} = 0$ (1)

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4}{2} \pm \frac{\sqrt{4} \times \sqrt{10}}{2}$$

$$x = 2 \pm \sqrt{10}$$

Since $k < 0$, the only solution is $x = 2 - \sqrt{10}$ . .

(1)

Question 15 continued

Question 15 continued

(Total for Question 15 is 7 marks)

16. A curve has equation $y = f(x)$, $x \geqslant 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants
- the curve has a stationary point at $(4, 3)$
- the curve meets the y-axis at -5

find $f(x)$, giving your answer in simplest form.

(6)

when curve is at stationary point, $f'(x) = 0$, $x = 4$ and $y = 3$.

$f'(x) = 4x + a\sqrt{x} + b$

$0 = 4(4) + a\sqrt{4} + b$

$0 = 16 + 2a + b$ (1) , $b = -2a - 16$ — (1)

To get $f(x)$, we will integrate $f'(x)$.

$f'(x) = 4x + a\sqrt{x} + b$ (1)

$f(x) = 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx + c$ (1)

y-intercept is -5, so the value of $c = -5$ (1)

$f(x) = 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx - 5$

from the stationary point, we know that $f(4) = 3$

$3 = 2(4)^2 + \frac{2}{3}a(4)^{\frac{3}{2}} + 4b - 5$

$3 = 32 + \frac{16}{3}a + 4b - 5$

$4b = -24 - \frac{16}{3}a$

$b = -6 - \frac{16}{12}a$ — (2)

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Question 16 continued

substitute ① into ②

$-2a - 16 = -6 - \frac{16}{12}a$

$-2a + \frac{4}{3}a = 10$

$-\frac{2}{3}a = 10$

$a = -15 \quad , \quad b = 14$ ①

$f(x) = 2x^2 + \frac{2}{3}(-15)x^{3/2} + 14x - 5$

$= 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$ ①

(Total for Question 16 is 6 marks)

17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

a) Let $q = 8$ and $p = 3$

$q^3 - p^3 = 8^3 - 3^3$

$= 485$ (1), which is a multiple of 5. Statement 1 is not true.

b) Let $p = 2n$ and $q = 2n+2$ (1)

$(2n+2)^3 - (2n)^3 = (2n+2)(2n+2)(2n+2) - 8n^3$

$= (4n^2 + 8n + 4)(2n+2) - 8n^3$

$= 8n^3 + 24n^2 + 24n + 8 - 8n^3$ (1)

$= 24n^2 + 24n + 8$ (1)

$= 8(3n^2 + 3n + 1)$ (1)

So, $q^3 - p^3$ is a multiple of 8.

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Question 17 continued

Question 17 continued

(Total for Question 17 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS